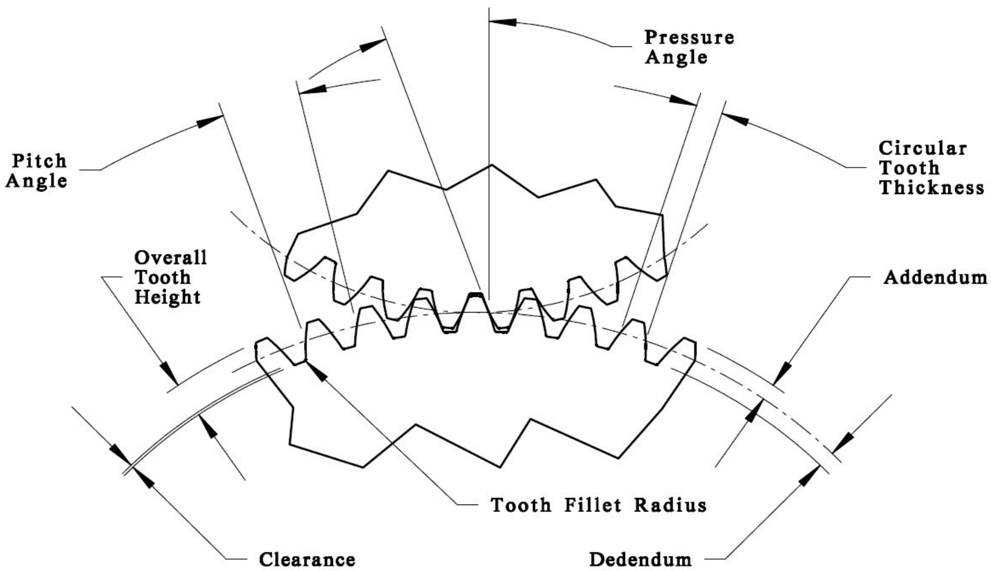
Basics of Spurgear Calculations



General Spurgear Nomenclature.

General Layout & Approximations:

In accordance with current standard (ANSI B6.1-1968/R1974), the base circle from which the involute of a gear is based is given by: $C_B = D_P * \cos(A_P)$; where C_B is the *base circle* diameter, D_P is the *pitch diameter* of the gear, and A_P is the *pressure angle* of the gear. An *involute* is the curve made as if "unwinding" the circumference of the base circle such that the "unwound part" is always (A) straight and (B) tangent to the base circle.

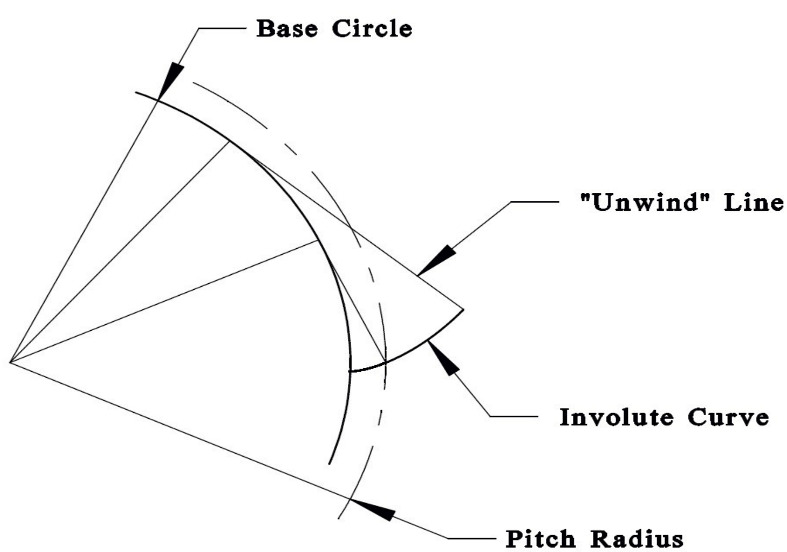


Figure of a Generated Involute.

Basics of Spurgear Geometry:

Most spurgears are defined by: *pressure angle* (A_P), *diametral pitch* (P_D) for inch-based gears or *module* (M_D) for metric gears, and the number of teeth (N). Traditionally, inch-based gears were limited to *pressure angles* of 14.5° or 20° while metric gears always had a 20° *pressure angle*. Recent years have allowed all types of gears to have *pressure angles* of 14.5°, 20°, or 25°. The greater the *pressure angle*, the stronger and noisier the gear train. The shallower the *pressure angle*, the weaker and quieter the gear train. Shallower *pressure angles* typically provide tighter fit allowances which reduce *backlash* and provide greater precision of relative shaft motions.

Geometric Calculations for Inch-Based Spurgearing:

In order for gear teeth to mesh properly, they must be the same size. In inch-based gears, that size is called out as *diametral pitch* (P_D). This is the *number of teeth* (N) divided by the *pitch diameter* (D_P) of the gear. I.E. $P_D = N/D_P$. The result must always be a multiple of ½ (.5). Standard *diametral pitch* sizes are: .5, **1.0**, 1.5, **2.0**, 2.5, 3.0, 3.5, **4.0**, 4.5, **5.0**, 6.0, 7.0, **8.0**, 9.0, **10.0**, 11.0, **12.0**, 14.0, **16.0**, 18.0, **20.0**, **24.0**, **32.0**, **48.0**, **64.0**, **72.0**, **80.0**, **96.0**, and **120.0**. Preferred sizes of *diametral pitch* are in boldface type.

Several things should be understood about *diametral pitch* (P_D). The first is that it is an inch-based construct. Second, the larger the *diametral pitch* value, the smaller the tooth. Third, overall radial or diametral height of the tooth (H_O) in any given *pitch* is constant. Fourth, the *pitch line width*, more often called the *circular tooth thickness* (T_{CR}), is also constant. This should be obvious in that, in order for the teeth to mesh, they need to have the same radial (or diametral) and circumferential (or peripheral) dimensions within each value for *diametral pitch*.

The table below is intended to provide an overall "yay-by-yar" feel for different *diametral pitch* sizes. I.E. a .5 P_D tooth will be 4.804 inches tall by 3.142 inches wide while a 10 P_D tooth will be .244 inches tall by .157 inches wide (etc.). The *preferred diametral pitch* values are shown in boldface type. Pictorial representations of *diametral pitch* tooth sizings may be found in catalogs such as: *Stock Drive Products/Sterling Instrument*, *Nordex*, and *Martin Sprocket & Gear.*

P_D	H_O	T_{CR}	P_D	H_O	T_{CR}
.5	4.8040 (in)	3.1416 (in)	11.0	.2222 (in)	.1428 (in)
1.0	**2.4040 (in)**	**1.5708 (in)**	**12.0**	**.2040 (in)**	**.1309 (in)**
1.5	1.6040 (in)	1.0472 (in)	14.0	.1754 (in)	.1122 (in)
2.0	**1.2040 (in)**	**.7854 (in)**	**16.0**	**.1540 (in)**	**.0982 (in)**
2.5	.9640 (in)	.6283 (in)	18.0	.1373 (in)	.0873 (in)
3.0	.8040 (in)	.5236 (in)	**20.0**	**.1240 (in)**	**.0785 (in)**
3.5	.6897 (in)	.4488 (in)	**24.0**	**.1040 (in)**	**.0654 (in)**
4.0	**.6040 (in)**	**.3927 (in)**	**32.0**	**.0790 (in)**	**.0491 (in)**
4.5	.5376 (in)	.3491 (in)	**48.0**	**.0540 (in)**	**.0327 (in)**
5.0	**.4840 (in)**	**.3142 (in)**	**64.0**	**.0415 (in)**	**.0245 (in)**
6.0	.4040 (in)	.2618 (in)	**72.0**	**.0373 (in)**	**.0218 (in)**

P_D	H_O	T_{CR}	P_D	H_O	T_{CR}
7.0	.3469 (in)	.2244 (in)	**80.0**	**.0340 (in)**	**.0196 (in)**
8.0	**.3040 (in)**	**.1963 (in)**	**96.0**	**.0290 (in)**	**.0164 (in)**
9.0	.2707 (in)	.1745 (in)	**120.0**	**.0240 (in)**	**.0131 (in)**
10.0	**.2440 (in)**	**.1571 (in)**			

Table of Diametral Pitch (P_D) vs. Overall Tooth Height (H_O) vs. Circular Tooth Width (T_{CR})

An alternative measure of tooth size is the *circular pitch* ($p_c = \pi/D_P$). Once the *diametral* or *circular pitch* have been determined, the rest of the values needed to define the gear may be found as:

Feature:	Diametral Pitch Basis:	Circular Pitch Basis:
Pitch Diameter (D_P):	$D_P = N/P_D$ (in)	$D_P = \pi/p_c$ (in)
Addendum (*a*):	$a = 1/P_D$ (in)	$a = p_c/\pi$ (in)
Dedendum (*d*):	$d = 1.25/P_D$ (in)	$d = 1.25p_c/\pi$ (in)
Clearance (*c*):	$c = .25/P_D$ (in)	$c = .25p_c/\pi$ (in)
Circular Tooth Thickness (T_{CR}):	$T_{CR} = \pi/2P_D$ (in)	$T_{CR} = p_c/2$ (in)
Tooth Fillet Radius (*r*):	$r = .15/P_D$ (in)	$r = .15p_c/\pi$ (in)
Gear OD (max):	$OD = (N + 2)/P_D$ (in)	$OD = (N + 2)p_c/\pi$ (in)
Angular Span of Tooth Set:	360/N (°)	360/N (°)
Angular Span of Individual Tooth:	360/2N (°)	360/2N (°)

Table of Gear Values for Inch-Based Gearing.

These equations provide the basis for spurgear geometry. Commercial gears are *hobbed* using precision-ground tools that control the actual *involute* geometry. Homemade or prototype spur-gears are often made with single-point *flycut* tools that only approximate the *involute* curve form. This is also the technique used to draft reasonably accurate representations of spurgears. A pair of radii, one starting at the *pitch circle* and moving out towards the *addendum* and the other one starting at the *pitch circle* and moving inwards towards the *dedendum* will approximate an actual *involute* curve if the following radius values are used:

Pressure Angle (A_P):	Dedendum Radius Value:	Addendum Radius Value:
14.5°	$11.857/P_D$ (in)	$6.532/P_D$ (in)
20°	$8.530/P_D$ (in)	$4.811/P_D$ (in)
25°	$6.740/P_D$ (in)	$3.765/P_D$ (in)

Table of Involute Approximation Radii for Inch-Based Gearing.

These are only approximations, but they work quite well in many instances. The text by Robert Porter, *Tool Grinding Attachment*, will be useful to anyone wishing to grind a single-point cutter for this purpose.

<u>Geometric Calculations for Metric Spurgearing:</u>

The metric equivalent of *diametral pitch* is the *module* (M_D). The *module* (M_D) multiplied by the *number of teeth* (N) equals the *pitch diameter* (D_P) in mm. It is the inverse of the *diametral pitch* rendered in mm. Standard values for *module* include: **0.3**, **0.4**, **0.5**, **0.8**, **1**, 1.125, **1.25**, 1.375, **1.5**, 1.75, **2**, 2.25, **2.5**, 2.75, **3**, *(3.25)*, 3.5, *(3.75)*, **4**, 4.5, **5**, 5.5, 6, *(6.5)*, 7, **8**, 9, **10**, 11, **12**, 14, **16**, 18, **20**, 22, **25**, 28, **32**, 36, **40**, 48, and **50**. The *module* values shown in boldface type are the *preferred module* values as defined in ISO-R53. The *module* values shown in parenthesis and italics are obsoleted values that were part of older *national standards* (DIN, JIS, etc.) that show up from time to time. The regular face *module* values are *secondary, non-preferred for new design* values.

Several things should be understood about the *module* (M_D). The first is that it is an millimetre-based construct. Otherwise it functions in the same general manner as *diametral pitch*, except that the larger the *module* value, the larger the tooth. A tooth of a given *module* will have the same overall *height*, *chord width*, and general shape as they must in order to mesh properly. In point of fact, the *module* **is** the gear's *addendum* (*a*) value.

The table below is intended to provide an overall “yay-by-yar” feel for different *module* sizes. I.E. a 0.5 M_D tooth will be 1.25 mm tall by 0.78 mm wide while a 10 M_D tooth will be 25 mm tall by 15.71 mm wide (etc.). The *preferred module* values are shown in boldface type. Pictorial representations of *module* tooth sizings may be found in catalogs such as: *Stock Drive Products/ Sterling International*, *Nordex*, and *Martin Sprocket & Gear.*

M_D	H_O	T_{CR}	M_D	H_O	T_{CR}
0.3	**0.75 (mm)**	**0.47 (mm)**	6	15 (mm)	9.42 (mm)
0.4	**1 (mm)**	**0.63 (mm)**	7	17.5 (mm)	11 (mm)
0.5	**1.25 (mm)**	**0.79 (mm)**	**8**	**20 (mm)**	**12.57 (mm)**
0.8	**2 (mm)**	**0.2 (mm)**	9	22.5 (mm)	14.14 (mm)
1	**2.5 (mm)**	**0.25 (mm)**	**10**	**25 (mm)**	**15.71 (mm)**
1.125	2.81 (mm)	1.77 (mm)	11	27.5 (mm)	17.28 (mm)
1.25	**3.13 (mm)**	**1.96 (mm)**	**12**	**30 (mm)**	**18.85 (mm)**
1.375	3.44 (mm)	2.16 (mm)	14	35 (mm)	21.99 (mm)
1.5	**3.75 (mm)**	**2.36 (mm)**	**16**	**40 (mm)**	**25.13 (mm)**
1.75	4.38 (mm)	2.75 (mm)	18	45 (mm)	28.27 (mm)
2	**5 (mm)**	**3.14 (mm)**	**20**	**50 (mm)**	**31.42 (mm)**
2.25	5.63 (mm)	3.53 (mm)	22	55 (mm)	34.56 (mm)
2.5	**6.25 (mm)**	**3.93 (mm)**	**25**	**62.5 (mm)**	**39.27 (mm)**
2.75	6.88 (mm)	4.32 (mm)	28	70 (mm)	43.98 (mm)

M_D	H_O	T_{CR}	M_D	H_O	T_{CR}
3	**7.5 (mm)**	**4.71 (mm)**	**32**	**80 (mm)**	**50.27 (mm)**
3.5	8.75 (mm)	5.5 (mm)	36	90 (mm)	56.55 (mm)
4	**10 (mm)**	**6.28 (mm)**	**40**	**100 (mm)**	**62.83 (mm)**
4.5	11.25 (mm)	7.07 (mm)	48	112.5 (mm)	70.69 (mm)
5	**12.5 (mm)**	**7.85 (mm)**	**50**	**125 (mm)**	**78.18 (mm)**
5.5	13.75 (mm)	8.64 (mm)			

Table of Module (M_D) vs. Overall Tooth Height (H_O) vs. Circular Tooth Width (T_{CR})

As with *diametral pitch* (inch-based) system gears, an alternative measure of tooth size is the *circular pitch* ($p_c = \pi M_D$). Once the *module* or *circular pitch* have been determined, the rest of the values needed to define the gear may be found as:

Feature:	**Module Basis:**	**Circular Pitch Basis:**
Pitch Diameter (D_P):	$D_P = NM_D$ (mm)	$D_P = \pi/p_c$ (mm)
Addendum (a):	$a = M_D$ (mm)	$a = p_c/\pi$ (mm)
Dedendum (d):	$d = 1.25M_D$ (mm)	$d = 1.25p_c/\pi$ (mm)
Clearance (c):	$c = 0.25M_D$ (mm)	$c = 0.25p_c/\pi$ (mm)
Circular Tooth Thickness (T_{CR}):	$T_{CR} = \pi M_D/2$ (mm)	$T_{CR} = p_c/2$ (mm)
Tooth Fillet Radius (r):	$r = 0.15M_D$ (mm)	$r = 0.15p_c/\pi$ (mm)
Gear OD (max):	$OD = (N + 2)M_D$ (mm)	$OD = (N + 2)p_c/\pi$ (mm)
Angular Span of Tooth Set:	360/N (°)	360/N (°)
Angular Span of Individual Tooth:	360/2N (°)	360/2N (°)

Table of Gear Values for Metric Gearing.

In practice, $M_D = 25.4/P_D$. This is one of those conversion factors of little use in the real world. The main use for this is to allow conversion of such things as the equivalent radius values we will use to approximate gears and render *inch-based* gearing into *Metric* for those trained in specific units (or vice versa). The table below provides a metric equivalent *Addendum* and *Dedendum* radius value set.

Pressure Angle (A_P):	**Dedendum Radius Value:**	**Addendum Radius Value:**
14.5°	$301.17*M_D$ (mm)	$165.91*M_D$ (mm)
20°	$216.66*M_D$ (mm)	$122.2*M_D$ (mm)
25°	$171.45*M_D$ (mm)	$95.63*M_D$ (mm)

Table of Involute Approximation Radii for Metric Gearing.

These are only approximations, but they work quite well in many instances. The text by Robert Porter, *Tool Grinding Attachment*, will be useful to anyone wishing to grind a single-point cutter for this purpose.

<u>Practical Application of Spurgear Values in Design:</u>

A full and detailed analysis of the strength of gear teeth and their safe application to machines upon which lives depend is **far** beyond the scope of this monograph. *The Standard Handbook of Machine Design* (Joseph Shigley and Charles Mischke) is the most commonly recognized source for the practice and application of this kind of analysis. *Handbook of Practical Gear Design* (Darle Dudley) is another commonly recognized source of such information. I find that the *Handbook of Gears* distributed **free** by Stock Drive Products/Sterling Instruments provides the equations for a more detailed analysis of common spurgear applications in a simple to use format.

First-Cut Analysis: In most spurgear applications, the weakest point of the drive occurs when a single tooth is carrying the transmitted torque. The area carrying that load will be the *circular tooth thickness* (T_{CR}) multiplied by the *width* (W) of the gear face (the thickness of the gear along the axis of the tooth). This area (T_{CR} * W) is then multiplied by either the allowable *shear strength* (for metallic gears) or the *flexural strength* (for plastic gears) to determine the allowable load this tooth may carry. MIL-HDBK-5 *Metallic Materials and Elements for Aerospace Vehicle Structures* is **the** "source authority" for minimum strength values for aerospace metals. The last revision that is publicly available is the J revision. This information is being **privatized** and you must get copies of it before it is removed from the *public domain*. Strength values for many materials is available from material manufacturers and suppliers. I find that Plastics International (http://www.plasticsintl.com/sortable_materials.php) has a very good list of properties that are tied to recognized ASTM test practices. Beware of properties listed as *typical* as they are often much too generous.

Assuming that you have kept your units consistent, we now have the total load we can expect the gear tooth to carry (T_{CR} * W * Allowable_Stress). This occurs at one-half of the *pitch diameter* (P_D) (this value is also known as the *pitch radius*) distance from the center of rotation. This gives us our **maximum force** and **radius** and, hence, our **maximum torque**. [As noted above, this is a very simplistic analysis.]

If you are working in inch-based units, then you have the torque in lb-in. The horsepower that can be transmitted by such a gear is given as:

$$HP = \tau * 63{,}024/rpm$$

Where HP is the horsepower capable of being transmitted, τ is torque (lb-in), and rpm is the shaft rotational speed of the gear in revolutions per minute. So long as the power transmitted through the gear is less than this calculated value, your design should be acceptable from a pure shear strength standpoint.

If you are working in metric units, then there are a couple of things to watch out for. The first is that gears are designed in mm. Thus, you got your area of stress as mm * mm as those are the units of metric gear design. Metric material strength properties are given in units of Pascals (N/m^2) – so you will typically need to divide your stress area by 1,000,000 to covert mm^2 to m^2. You will also need to divide your *pitch radius* value by 1000 to convert mm to m. Power in the metric system is measured in Watts (N-m/sec). Thus, the maximum power that may be carried by a gear is given as:

$$W = \tau * 9.459/rpm$$

Where W is the power in Watts capable of being transmitted, τ is torque (N-m), and rpm is the shaft rotational speed of the gear in revolutions per minute. So long as the power transmitted through the gear is less than this calculated value, your design should be acceptable from a pure shear strength standpoint.

Additional Factors to Consider: The bending of the tooth itself under the load is a non-trivial variable section beam analysis. The greater the *pressure angle* (A_P) of the gear, the stronger the tooth is in bending. The smaller the *pressure angle* (A_P) of the gear, the weaker the tooth is in bending. Similarly, poor material or finish choices for a spurgear may reduce the load-carrying capacity of the tooth by 50%. Additionally, improper lubrication or high friction values may allow gear to heat to the point where they fail. If any of these factors appear to influence the function of your gear train, refer to the handbooks cited above. The application is beyond the scope of this monograph.

Liability Waiver Statement: I am a registered Professional Engineer. I have done **no** study or analysis of anyone's application of the material presented herein. I am providing this information to aid people who wish to use spurgears in hobby-related applications where a failure is only going to ruin some hours of work done for fun. **Anyone applying this information where life or limb is at stake is violating the provision of this information. I refuse liability for any damages resulting from the use of this information.** I make the assumption that anyone who is interested in the information provided herein has the basic knowledge and skills to apply spur-gear technology to their project. If the information provided herein does **not** make sense to you, the **do not use it**. If you are **not** willing to accept these conditions, do **not** use the information I have provided!

Finally, *Tool Grinding Attachment* by Robert Porter is a 20 page booklet usually available from Lindsay Publications. Mr. Porter is a watchmaker/repairman who has published several booklets on the tools of his trade. You can reach Lindsay Publications at: http://www.lindsaybks.com. My experience with Lindsay Publications is been pretty good, though you have to look carefully at the books they sell. They do have quite a few “UFO Science” type books.